On a quadratic conservative Vlasov–Maxwell scheme toward large-scale kinetic simulations

T. Shiroto[1,2], Y. Sentoku[1], N. Ohnishi[2]

[1] Institute of Laser Engineering, Osaka University, Suita, Japan
[2] Department of Aerospace Engineering, Tohoku University, Sendai, Japan

Since a particle-in-cell (PIC) method born in 1950's, most of theoretical plasma scientists have suffered from a problem so-called "numerical heating." This is a numerical phenomenon which increases the total energy in a simulation domain until the local Debye length is close to a grid size even if there are no energy sources. Therefore, a grid size should be small enough to stabilize the numerical heating, resulting in a small computational domain and short computational time. The problem is given by the fact that the conservation laws of momentum and energy are described in the non-conservative formulation. To overcome this issue, some mathematical investigations have been performed [1,2], but there are no schemes that can maintain the conservation laws of momentum and energy simultaneously. On a Vlasov simulation, which directly discretizes the Vlasov–Maxwell system, a charge-momentum-energy conserving scheme has been developed [3]. However, this scheme depends on a spectral method which cannot accept non-periodic boundary conditions. Thus, we cannot use this algorithm to analyze a large-scale laser-plasma interaction.

Recently, we developed a quadratic conservative Vlasov–Maxwell scheme based on a finite-difference method, which strictly preserves the conservation laws of charge, momentum, and energy [4]. We derive these conservation laws, the Gauss's law, and the solenoidal constraint of magnetic field from the relativistic Vlasov–Maxwell system using the product rule, integration-by-parts, and commutative laws of partial-differential operators. The key strategy of our scheme is to strictly maintain these mathematical formulae in discrete form. Numerical experiment of a relativistic Weibel instability suggests that the quadratic conservative scheme preserve the conservation laws of charge, momentum, and energy only with round-off errors. The linear growth rate of the relativistic Weibel instability [5] is well reproduced. This work was supported by JSPS Grant-in-Aid for scientific research KAKENHI (15J02622, 15K21767).

References

[1] G. Chen, L. Chacón and D.C. Barnes, J. Comput. Phys. 230, 7018 (2011).
[2] S. Markidis and G. Lapenta, J. Comput. Phys. 230, 7037 (2011).
[3] G.L. Delzanno, J. Comput. Phys. 301, 338 (2015).
[4] T. Shiroto, N. Ohnishi and Y. Sentoku, arXiv, 1802.07238 (2018).
[5] H.H. Kaang, C.-M. Ryu and P.H. Yoon, Phys. Plasmas 16, 082103 (2009).